

Exercice 1 :

a. $z_1 + z_2 = 1 + 2i + (-3 + 4i) = 1 + 2i - 3 + 4i = -2 + 6i$

b. $z_1 z_2 = (1 + 2i)(-3 + 4i) = -3 + 4i - 6i + 8i^2 = -3 - 2i - 8 = -11 - 2i$

c. $4z_1 - iz_2 = 4(1 + 2i) - i(-3 + 4i) = 4 + 8i + 3i + 4 = 8 + 11i$

d. $\frac{z_2}{z_1} = \frac{(-3 + 4i) \times (1 - 2i)}{(1 + 2i) \times (1 - 2i)} = \frac{-3 + 6i + 4i - 8i^2}{1^2 + 2^2} = \frac{5 + 10i}{5} = 1 + 2i$

e. $(z_1)^2 = (1 + 2i)^2 = 1 + 4i + 4i^2 = -3 + 4i$

f. $\frac{z_1 - 6i}{z_2 + 5 - 6i} = \frac{(1 - 4i) \times (2 + 2i)}{(2 - 2i) \times (2 + 2i)} = \frac{2 + 2i - 8i - 8i^2}{2^2 + 2^2} = \frac{10 - 6i}{8} = \frac{5 - 3i}{4}$

Exercice 2

1. $\bar{z} = \overline{(1 - 2i)(3 + i) - 5 + 4i} = (1 + 2i)(3 - i) - 5 - 4i$

2. $\bar{z} = (1 + 2i)(3 - i) - 5 - 4i = 3 - i + 6i - 2i^2 - 5 - 4i = i$

3.

Exercice 3

$$2z - i = 3iz - 2 \Leftrightarrow 2z - 3iz = i - 2 \Leftrightarrow (2 - 3i)z = i - 2$$

a) $\Leftrightarrow z = \frac{i - 2}{2 - 3i} = \dots = \frac{-7 - 4i}{13}$

b)

$$i\bar{z} - 4 = 2i - 3\bar{z} \Leftrightarrow i\bar{z} + 3\bar{z} = 2i + 4 \Leftrightarrow (i + 3)\bar{z} = 2i + 4$$

$$\Leftrightarrow \bar{z} = \frac{2i + 4}{i + 3} = \dots = \frac{7 + i}{5}$$

Donc $z = \frac{7 - i}{5}$

c)

$$(2 + i)\bar{z} = 4z - 6i \Leftrightarrow (2 + i)(a - ib) = 4(a + ib) - 6i$$

$$\Leftrightarrow 2a - 2ib + ia + b = 4a + 4ib - 6i$$

$$\Leftrightarrow \underbrace{2a + b}_{x} + \underbrace{i(a - 2b)}_{+iy} = \underbrace{4a}_{x'} + \underbrace{i(-6 + 4b)}_{+iy'}$$

$$\Leftrightarrow \begin{cases} 2a + b = 4a \\ a - 2b = -6 + 4b \end{cases} \Leftrightarrow \begin{cases} b = 2a \\ a - 12a = -6 \end{cases} \Leftrightarrow \begin{cases} b = \frac{12}{11} \\ a = \frac{6}{11} \end{cases}$$

Donc $S = \left\{ \frac{6}{11} + \frac{12}{11}i \right\}$

d) $\Delta = -4 < 0$ donc 2 solutions complexes $z_1 = \frac{2 + 2i}{2} = 1 + i$ et

$$z_2 = \frac{2 - 2i}{2} = 1 - i$$